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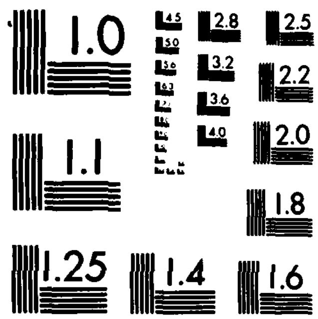
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Research Report CCS 451

A COMPARATIVE STUDY OF DATA ENVELOPMENT ANALYSIS AND OTHER APPROACHES TO EFFICIENCY EVALUATION AND ESTIMATION†

by

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Abstract

Data Envelopment Analysis (DEA), a new methodology based on mathematical programming models, provides an approach to evaluation of the relative efficiency of organizations, especially not-for-profit organizations which have multiple outputs and inputs. This paper uses an artificial data base to evaluate DEA relative to other alternatives such as ratio and regression analyses. The results of this study generally favor DEA not only for identifying inefficiencies but also for locating their sources and estimating their amounts in particular DMUs (Decision Making Units). Statistical regressions performed very poorly, per se, as well as by comparison. Reasons for the poor performance of these customary statistical regression approaches are indicated along with possible ways of improving this performance.

Key Words

Efficiency
Hospitals
Data Envelopment Analysis
Statistical Regression
Ratio Analysis
Simulation
Validation

1. Introduction

Data Envelopment Analysis (DEA) is a new efficiency measurement methodology developed by A. Charnes, W. W. Cooper, and E. Rhodes as set forth in [12] [13] and [14].1/ It is designed to measure the relative efficiency of Decision Making Units (DMUs) which use multiple inputs to produce multiple outputs even when the underlying production function is not known and where, additionally, these functions may also be multiple in character. This contrasts with the situation for statistical techniques and theory, e. g., as employed in economics, where either the underlying production function must be known, or at least its parametric form must be assumed before it can be used to evaluate efficiencies and where, usually, a single functional form is also assumed. See, e. g., Feldstein [18]. See also [32] and [33]. The latter, regression approaches, are thus limited, especially in the case of public sector institutions such as hospitals, etc., where programs and activities are even less readily identified for such assumptions than is the case in industrial production.

DEA has now been applied to several types of organizations including education [5] [6], health care [4] [29], Navy recruiting [22], and criminal court systems [21]. Nevertheless something more is required and, in particular, the validity and reliability of DEA in locating inefficient DMUs, identifying the inputs (and/or outputs) where the inefficiencies occur and estimating their amounts or magnitudes all need to be evaluated. One way to approach this task is via a situation in which the identity of the truly inefficient units is known along with the sources and amounts of this inefficiency. This paper therefore attempts to evaluate DEA through use

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1/ See also [25].

of an artificial data base where the efficient and inefficient DMUs are all known in numerical detail. DEA's performance is then compared with other commonly employed techniques such as ratio and regression analyses.

Regression and ratio analyses were selected for these evaluations because they are widely used in fields like health services, which is the field we shall use to guide our data base construction. In this paper we restrict our examination only to some of the fairly simple forms of ratio and/or regression approaches that are in wide use.1/ More sophisticated regression techniques such as the translog function and other so-called "flexible functional form" approaches are considered elsewhere. See Sherman [29].

The following section describes how the data base was constructed and section 3 discusses the data base that was developed. Section 4 describes the version of DEA that will be used while sections 5, 6 and 7 discuss the results of applying DEA, ratio and regression analyses to this data base. The resulting comparisons are summarized in section 7 with respect to the ability of these techniques to identify and distinguish between efficient and inefficient DMUs. Section 8 then extends the uses of DEA to locating and estimating the amounts of inefficiencies in particular DMUs in ways that are not generally available when the ratio or regression approaches are used. A concluding section then discusses some of the shortcomings found in these other approaches and indicates where they differ from DEA and how some of their shortcomings might be repaired.

1/ Similarly, only one version of DEA is used and no attempt is made to distinguish between various types of efficiencies such as scale vs. technical efficiencies and other sources of inefficiency such as are examined in [3]. Finally, we did not use statistical principles of experimental design, such as randonization replication, etc. to develop our data base, as was done in [2], and hence can make only limited use of statistical significance tests and like devices for generalizing our results. Our purpose is rather to supply insight of potential value on the use of the techniques we study rather than to secure generalizations for the different data situations that might be encountered in actual practice. See concluding section of this paper. See also [2] where similar conclusions are reached in experiments conducted in accordance with the usual principles of the statistical design of experiments.

2. Model Structure and Data Generation

The artificial data set was constructed by defining a hypothetically "known" technology which applies to all Decision Making Units (DMUs) and defines efficient input-output relationships for each of them.[1/] Inefficiencies which were explicitly introduced for certain DMUs take the form of excess inputs used for the output levels attained. Hence, a DMU that achieves its output level by using only the amount of inputs required by this hypothetical technology is efficient while a DMU that uses more than the required amount of any input is inefficient. To make the inputs and outputs easier to recognize, they are referred to and labelled in the context of a hospital study as one area of potential interest. See Sherman [29]. We assume that these hospitals are all public (not-for-profit) institutions so that the usual profit calculus and/or price-weighted reductions to a scalar measure of efficiency evaluation are not wholly appropriate.

1/ Knowledge gained from the study of Massachusetts hospitals reported in [29] was used in the choice of inputs and outputs and in the construction of the data set.

The set of artificial hospital data generated for our simulation consisted of three outputs produced with three inputs during a one year period of time[1/] as follows:

Outputs		Inputs	
y_1 :	Regular patient* care/year (patients treated in one year with average level of inputs for treatment)	x_1:	Staff utilized in terms of full-time equivalents, i.e., (FTE s)/year
y_2 :	Severe patient* care/year (patients treated in one year with severe illness requiring higher input levels than regular patients for more complex treatment).	x_2:	Number of hospital bed days available/year
y_3 :	Teaching of residents and interns/year (number of individuals receiving one year of training)	x_3:	Supplies in terms of dollar cost/year

*measured in terms of number of patients treated

The data set to be generated was for 15 hypothetical hospitals which we label as H1, H2, ..., H15, to represent the pertinent DMUs.[2/] They are all assumed to achieve their outputs via a common production process, which they may use efficiently or inefficiently. The resulting observed values are then constructed in a manner that we shall shortly describe.

In this study we shall focus on input inefficiencies, by which we mean that one or more of the above inputs may be used in excess to obtain a particular hospital's output values. Although we could also similarly study output deficiencies (in the form of output shortfalls from given inputs)[3/] we shall not lengthen the paper to undertake that study here. In any case the known values of the per unit inputs for efficient production are given in Exhibit 1 inserted at the end of this paper.

1/I. e., we are considering all data as annual rates.

2/Subdivisions may also be used such as, e. g., the surgical units within each hospital that were studied in [29].

3/An output shortfall approach from given inputs is used in [2].

The usual regression approach to efficiency and related types of economic analyses in multiple output situations uses a single aggregate function of a linear or logarithmic variety in which total cost is regressed against the observed output values. See, e. g., [18]. This approach carries with it a variety of assumptions[1/] which we shall try to favor in our construction by using the same prices and a common technology for all DMUs. We shall not assume that all DMUs operate on their efficiency frontiers, however, but we shall otherwise proceed in accordance with the usual methods of estimation, testing and analyses that have been commonly employed in regression studies of health services and related fields.

To make the sense of this discussion more precise, we present our expressions for generating the inputs required for efficient operations by any hospital in the following form:

$$x_{ij} = \sum_{r=1}^{3} a_{irj}\, y_{rj} \qquad (1)$$

where

x_{ij} = amount of input i used per year by hospital j

y_{rj} = amount of output r produced per year by hospital j

a_{irj} = amount of input i used per unit of output r by hospital j during the year.

1/ See, e. g., Sato [27].

2/ A use of DEA to distinguish coefficients for input-output analyses derived from data for efficient and inefficient sets of operations may be found in Schinnar [28].

These a_{irj} values, which are fixed constants, represent an efficient set of coefficients which may be used to generate the inputs required for any observed (or planned) level of outputs. In some cases we will assign values $\hat{a}_{irj} > a_{irj}$ for some i, r and j to represent managerial (= hospital) inefficiencies which yield values

$$\hat{x}_{ij} = \sum_{r=1}^{3} \hat{a}_{irj} \; y_{rj} , \tag{2}$$

with $\hat{x}_{ij} > x_{ij}$ when inefficiencies are present.

The efficient a_{ir} values are given, free of any of the $j = 1,\ldots, 15$ hospital identification subscripts, in Exhibit 1. These values are the same for all hospitals so that $a_{11} = .004$ FTE/patient represents the efficient labor requirement in Full Time Equivalent units per regular patient. Similarly $a_{12} = .005$ FTE/patient represents the efficient requirement for a severe patient and $a_{13} = .03$ FTE/training unit represents the efficient requirement to train one new resident/intern during a year.

Analogous remarks apply to the values $a_{21} = 7$ bed days/patient, and $a_{22} = 9$ bed days/patient for regular and severe patients, respectively, shown in the Bed Days column of Exhibit 1. The blank shown in the row for Training Units in this column means that $a_{23} = 0$ applies. That is, no Bed Days enter into the training outputs.

Finally, $a_{31} = \$20$/patient and $a_{32} = \$30$/patient represent the efficient level of supplies required per regular and severe patients, respectively, while $a_{33} = \$500$/training unit is the coefficient for efficient training operations in output $r = 3$. Putting this $i = 3$ input in dollar units avoids the detail that would otherwise be needed to identify the different types of supplies that would be required for teaching and for different types of patient treatments.

DEA does not require reductions to cost equivalents. The various outputs and inputs may be specified in different units of measure and, indeed, it can be shown that the resulting DEA efficiency value is independent of the units of measure used in any output or input.1/ On the other hand reductions like these are required for the ratio and regression measures we shall also study. Therefore we next show how the efficient costs are derived to obtain this part of our data set. This is done via expressions of the form,

$$c_r = \sum_{i=1}^{3} k_i a_{ir} \qquad r = 1, 2, 3, \tag{3}$$

where we have omitted the index j for hospital identification because only efficient costs are being considered. Here k_i represents the cost of the i^{th} input requirement for the r^{th} output under efficient operations where

$$\begin{aligned} k_1 &= \$10{,}000/\text{FTE} \\ k_2 &= \$10/\text{bed day} \\ k_3 &= \$1/\text{supply unit.} \end{aligned} \tag{4}$$

These data are then combined with the preceding a_{ij} values to obtain

$$\begin{aligned} c_1 &= k_1a_{11} + k_2a_{21} + k_3a_{31} = \$130/\text{regular patient} \\ c_2 &= k_1a_{12} + k_2a_{22} + k_3a_{32} = \$170/\text{severe patient} \\ c_3 &= k_1a_{13} + k_2a_{23} + k_3a_{33} = \$500/\text{training unit.} \end{aligned} \tag{5}$$

These are the formulas used at the bottom of Exhibit 1 to produce the efficient cost of outputs shown in the last column in the body of the table.

1/ Provided, of course, that these same units of measure are used for the specified output (or input) in the data for every DMU. See Charnes, Cooper and Rhodes [11] and [12]. See also Rhodes [25].

3. Data Base Development

We now turn to Exhibit 2 which reflects the composition of inefficient and efficient hospitals included in our data base. The hypothesized "actual" (or observed) inputs per unit output used by each hospital, whether efficient or not, are listed in Exhibit 2, columns 9-16 with inefficient input levels per unit of output denoted by ( ). Column 17 reflects the actual vacancy rate (% of unused bed days available during the year) where, as noted in Exhibit 1, an efficient hospital is expected to have a 5% vacancy rate.

We develop the actual inputs used for each hospital in the manner we have already described by first selecting an arbitrary set of output values for each of the hospitals listed in the left-hand stub.[1/] Teaching units per year are reflected in column 6, regular patients treated during the year are in column 7, and severe patients treated during the year are in column 8.

Other ways of summarizing patient care outputs for later use are included in columns 4 and 5. Column 4 reflects total patients as the sum of column 7 and column 8. Column 5 reflects the percentage (%) of severe patients treated which is based on (column 8) ÷ (column 4) X (100). We develop this percentage output measure because it reflects output data in a form which is often used to evaluate efficiency in many real data sets.[2/]

The inputs used by each hospital to produce the outputs in columns 6, 7, and 8 are reflected in columns 1, 2, and 3. Column 1 contains the full time equivalents (FTE s) of labor years used. Column 2 has the bed days/year which were available and column 3 gives the supply dollars used during the year.

1/ Although these values could have been selected by statistical principles--e.g., of an experimental design variety--there seemed to be little point in doing so because our objective was to secure insight rather than the kinds of generalizability that require statistical tests of significance. See [2], however, for a study of the latter type.

2/ *See the discussion in Sherman* [29].

The values in columns 1, 2, 3 reflect mixtures of efficient and inefficient utilization of resources because of the way they were derived. We can clarify this by means of Exhibit 3 which illustrates how the data for H1, an efficient DMU, and H15, an inefficient DMU, were constructed. H1 is efficient and therefore used the same inputs per unit outputs as the structural model in Exhibit 1. During the year, H1 provided care for 3000 regular patients, 2000 severe patients, and 50 training units of service. It therefore utilized (.004)(3000) + (.005)(2000) + (.03)(50) = 23.5 FTE s in that year. H15 produced the same outputs as H1 but was inefficient in its use of certain inputs. It used .005 FTEs /regular patient, while it adhered to the structural model FTE usage rates for severe patients (.005 FTEs /patient) and training (.03 FTEs /training unit). H15 therefore used (.005)(3000) + (.005)(2000) + (.03)(05) = 26.5 FTEs /year to produce the same outputs. Similarly, H15 is <u>inefficient</u> in the number of bed days used and supply dollars used per regular patient but is <u>efficient</u> in the amount of bed days and supply dollars consumed for severe patients and for supply dollars used for teaching outputs. Bed days and FTEs and supply dollar inputs are also calculated in Exhibit 3 to further illustrate the way the data base was constructed.

The number of FTEs , bed-days, and supply dollars inputs were calculated as illustrated in Exhibit 3 for each hospital based on the arbitrarily assigned output mix of regular patients, severe patients and training units and actual efficient or inefficient input per unit output rate reflected in Exhibit 2.

Certain relationships posited in the structural model are generally not known, like the actual amount of staff time and supplies that are required to support each intern or resident at a hospital. We nevertheless explicitly introduce these relationships to determine if the efficiency measurement techniques we will apply can uncover them. Before proceeding, however, it should perhaps be noted that when the underlying structural model is known, the determination of which DMUs are inefficient can be directly determined and techniques such as we will be considering would be unnecessary for purposes of efficiency evaluation.

4. The DEA Model:

The Charnes Cooper Rhodes (CCR) model for data envelopment analysis which we will use assumes the following form:

Objective:

$$\max h_o = \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{i=1}^{m} w_i x_{io}}$$

Constraints: (6)

Less than Unity Constraints:
$$1 \geq \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} w_i x_{ij}} \; ; \; j = 1,\ldots,15$$

Positivity Constraints:
$$0 < u_r \; ; \; r = 1,\ldots,s$$
$$0 < w_i \; ; \; i = 1,\ldots,m$$

Data:

Outputs: y_{rj} = observed amount of r^{th} output for j^{th} hospital

Inputs: x_{ij} = observed amount of i^{th} input for j^{th} hospital.

1/ Other models which might have been used can be found in [3] and [15]. See also [16].

This model is therefore in fractional programming form with fractional constraints. As noted in Charnes, Cooper and Rhodes [13] it may be replaced by an ordinary linear programming model that also has non-Archimedean conditions imposed on the variables for what are here referred to as positivity constraints.1/

We shall not enter into this kind of development but shall instead try to explicate what is happening in our DEA analysis by means of the above model. First we observe that the efficiency ratings are all restricted to an upper limit of unity. One of the $j = 1,\ldots,15$ hospitals, when singled out for efficiency evaluation, is represented in the objective as well as the constraints. By virtue of the latter condition we must have $\max h_o = h_o^* \leq 1$. Furthermore all observations y_{ij} and x_{ij} are positive so that, together with the positivity imposed on the variables, we will also have $0 \leq h_o^* \leq 1$ with $h_o^* = 1$ when and only when DMU_o, the DMU being evaluated, is efficient.

Qualifications need to be entered to allow for the presence of slack in the corresponding linear programming model.2/ We will not treat this topic in rigorous detail in the present paper but will instead supply an illustration with accompanying discussion that will provide insight into what is involved. Here we need only say that when slack is present in some input then, with efficiency, that input may be reduced to a new input level by

1/ See Charnes, Cooper, Lewin, Morey and Rousseau [10] for a precise development.

2/ Any slack which occurs in (6) is simply the complement of an efficiency rating but the development in [10] provides a way of identifying the presence of non-Archimedean values in (6) with slack in the corresponding linear programming model.

removing the slack without affecting any output or any other input. Hence the input which involved this slack was excessive and the operation could not have been efficient.

Bearing this in mind we next initiate our DEA analysis by reference to the data of Exhibit 2 after which we shall attempt to compare the resulting efficiency ratings with cost ratio and regression approaches applied to this same data base.

5. Applications to Artificial Data Base.

Applying (6) to Exhibit 2 with each of H1,...,H15 inserted in the objective produces the h_o^* values reported in Table 1. Every one of the efficient DMU's has received a rating of $h_o^* = 1$ but two inefficient DMU's--H10 and H13--are also accorded a value of $h_o^* = 1$ even though they are inefficient. The six DMU's that are rated as inefficient, with $h_o^* < 1$, are accorded these values by comparison with certain efficient units that comprise an efficiency reference set for the inefficient DMU (see Table 1). For example, H8 was found to be inefficient by direct comparison with H4; and H15 is being compared directly with H4, H6, and H7. This reference set, we need only note here, is supplied as part of the optimum basis in the linear programming computations. Hence the model and computing routines supply what is wanted without extra effort and, furthermore, the appearance of a DMU as part of an optimal basis ensures that it is efficient so that separate computations need not be made for these entities if that is all that is wanted.[1/]

1/ Computer codes are available for effecting these computations. See [6]. New software by I. Ali and J. Stutz is also available from the Center for Cybernetic Studies at The University of Texas at Austin which detail the efficient facets observed.

It might be observed that the two inefficient DMU's that were accorded efficiency values of $h_o^* = 1$ have no such reference sets. This suggests that they have special properties which can be submitted to further analysis by means of the non-Archmidean formulations that we touched on earlier in the text.1/ We shall not turn aside to deal with that topic. Instead we shall simply accept this identification of H10 and H13 as a possible weakness of DEA in the comparisons we are making with other techniques since (as in this case) it can happen.

1/Note also that neither H10 nor H13 enter into the reference set for any other DMU.

Table 1

Efficient DMU's	DEA Efficiency Rating (E)	Efficiency Reference Set
H1	1.0	
H2	1.0	
H3	1.0	
H4	1.0	
H5	1.0	
H6	1.0	
H7	1.0	

Inefficient DMU's	DEA Efficiency Rating (E)	Efficiency Reference Set
H8	0.99	H4
H9	0.98	H1, H2, H6
H10	1.0	
H11	0.85	H4, H7
H12	0.99	H1, H4, H6
H13	1.0	
H14	0.99	H1, H4, H6
H15	0.87	H4, H6, H7

6. Cost Ratio Analysis

We now consider how a manager, e. g., in a rate setting commission for some state,[1/] might determine which DMUs are more and less efficient when using ratios, a widely used form of analysis to evaluate financial and operating performance. In this example, all the inputs are jointly used by these DMUs to produce three outputs so that we cannot proceed as we might in the single output case. A number of different ratios might be developed to evaluate different sets of relationships such as FTEs/patient, FTEs/severe patient, FTEs/regular patient, FTEs/teaching output, bed days/patient, bed days/severe patient, etc. Such a set of ratios does not explicitly recognize the joint use of these inputs to produce these various outputs. In addition, for the set of ratios calculated, a DMU may be among the highest (least efficient) for certain ratios and lowest (most efficient) for other ratios. This leads to some ambiguity as to whether that DMU is efficient or inefficient and calls for some method of weighting or ordering the importance of the ratios to gain some overall assessment of efficiency such as was generated using DEA in Table 1.

Rather than address this issue directly, we will focus on a type of unit costing ratio analysis that is often applied to hospitals and other organizations to evaluate DMU performance. By design we can say that all 15 hospitals (DMUs) paid the same price per unit for each type of input and thus ignore possible difficulties which arise for a ratio analysis when this is not the case. That is, we can combine the inputs into dollar units without the confounding effect of differing input costs. Rather

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1/ For instance, see [23] and [24].

than deal with all these outputs, the teaching output might be viewed as a by-product or secondary output and the patients might be viewed as a single output rather than segregate this into different categories of severity. This simplifying procedure is not wholly defensible from a cost accounting standpoint. Nevertheless, in the absence of any other way of combining and weighting the outputs, similar approaches have been used for hospitals as well as other types of DMUs (see for example [23]), and this is the way we shall proceed.

Table 2 column (A) reflects the average cost per patient for each DMU. This results in a ranking of hospitals reflected by the parenthesized number directly to the right of the average cost figure in Table 2. The lowest cost (most efficient) DMU is ranked 1 and highest cost (least efficient) DMU is ranked 13. This ranking erroneously classifies H13 (ranked 6) as more efficient than H3 (rank 7) and H6 (rank 9) and it classifies H9 as more efficient than H6. In addition, there is no objective means for determining the cutoff cost level to segregate efficient and inefficient units.

If the efficient relative costs of certain outputs are known, the outputs can be weighted to reflect a cost per weighted unit of output. In this case we know the efficient cost of a regular patient ($130) and a severe patient ($170) and the patient units can therefore be weighted to value each severe patient as the equivalent of 170/130 ~ 1.3 regular patients. For example, H1 would have adjusted patient output units of 3000 regular patients + 2000 x 1.3 severe patients for an adjusted total of 5600 patients. Dividing this patient total into $775,500, the total cost for H1 shown in Exhibit 3, results in $138.48, the case mix adjusted average cost shown for H1 in column (B) of Table 2.

Table 2

Single Output Measures

Hospital Efficient Units	Average Cost per Patient (A)		Case Mix Adjusted Average Cost per Patient (B)		Case Mix Adjusted Average Cost per Patient Segregated into High and Low Levels of Teaching Outputs: Low* (C)		High* (D)	
H1	$155.10	(2)	$138.48	(4)	$138.48	(2)		
H2	163.32	(5)	138.40	(3)	138.40	(1)		
H3	168.32	(7)	142.65	(8)			$142.65	(3)
H4	160.10	(4)	142.94	(9)	142.94	(6)		
H5	158.38	(3)	137.73	(2)			137.73	(2)
H6	170.15	(9)	140.12	(5)	140.12	(3)		
H7	142.60	(1)	135.81	(1)			135.81	(1)
Inefficient Units								
H8	176.95	(11)	157.99	(12)**			157.99	(6)
H9	168.32	(7)	142.64	(7)	142.64	(5)		
H10	169.69	(8)	161.61	(14)**			161.61	(7)
H11	170.33	(10)	153.10	(10)	153.10	(7)		
H12	178.33	(12)	155.07	(11)			155.07	(5)
H13	165.68	(6)	142.00	(6)	142.00	(4)		
H14	178.33	(12)	155.07	(11)			155.07	(5)
H15	179.74	(13)	160.48	(13)**	160.48	(8)		
Mean	167.02		146.94		144.77		149.42	
Standard Deviation			8.82		7.36		9.66	

* Low teaching outputs were 50 units and high teaching outputs were 100 units as per Exhibit 3, Col. 6.

**Hospitals more than one standard deviation over average cost.

The adjusted cost per patient is reflected in column (B) of Table 2 with the new ranking in parenthesis immediately to the right of the average cost per day. Even with this (normally not available) weighting of patients we continue to have a misranking with inefficient DMUs H9 and H13 being ranked as more efficient than H3 and H4. If we further segregate the 15 DMUs by the third output (teaching), as is sometimes done, and separate them based on those with high (100 units) versus low (50 units) teaching outputs, the ranking based on unit costs is reflected in columns C and D in Table 2. At this point, we have achieved an accurate ranking for the high teaching output hospitals but we still have not achieved an accurate ranking for the low ones. Because we have only two values for these outputs, at 50 and 100 "teaching units," we could distinguish high vs. low output hospitals fairly easily in the present case, but generally there will be many more values to consider with no objective guidance available for separating high from low teaching output values and the difficulty of distinguishing efficient from inefficient DMUs will then be compounded.

The problem of locating a point beyond which DMUs are considered inefficient is typically addressed by establishing a subjective cutoff value, even though there is no assurance, theoretical or otherwise, that the inefficient units will be accurately located through this process. For example, if the cutoff was set at one standard deviation above the mean adjusted cost per patient, only 3 DMUs (H8, H10 and H15) would be identified as inefficient as indicated in column (B) of Table 2.[1/]

The DEA ratings in Table 1 do not lend themselves to rankings of the kind used in Table 2. As will be seen below, these efficiency measures

1/At $0.6745\sigma = 5.95$, three more DMUs (H11, H12 and H14) would be added to this inefficient set. We record this as an additional possibility for improving this kind of identification even though most of the commonly used adjustments are in the direction of $k\sigma$, with $k > 1$.

are intended to supply estimates of excessive resource utilization relative to the Efficiency Reference Sets from which these ratings are derived. If, on the other hand, one uses the estimated resource savings as a basis and accords the same ranks to DMUs with equal efficiency ratings, a more informative set of ranks would be available from Table 1 than Table 2.[1/] Whether ranked or not, however, Table 1 is more informative than Table 2 provided, of course, that the efficiency values exhibited in Table 1 are reasonably accurate.

7. <u>Regression Analysis</u>

In industries, including the "health industry," where the efficient input-output technology is ambiguous or at least is not known with any real precision, regression analysis has been applied in order to gain "insights" into the production relationships that might underlie the observations that have been generated from past utilization of these processes. There are, of course, a variety of problems that are encountered when using traditional regression analyses to evaluate the efficiency of individual DMUs. One problem in most such studies is that one relatively smooth relation is posited to obtain the parameter estimates that are needed. Another problem is that the estimated parameter values are based on least squares estimates which

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1/ In general one would also need to impute dollar magnitudes or other weights to the potential savings.

provide "mean" or "central tendency" values that reflect a mixture of efficient and inefficient behavior in the data set.[1/] Thus, even if the posited functional forms are correct, the estimated regressions will only reflect efficient relationships if all units in the study are themselves efficient. Whatever reasons may be used to justify such assumptions in competitive industries, they are likely to be much weaker in not-for-profit settings such as education, health, and government.

Nevertheless such approaches have been extensively employed and so we now consider the extent to which regression analysis as it has been used, e. g., in health studies, might be employed to identify the inefficient units in the artificial data set. In the process we shall also locate other potential problems in the use of such analyses even when we can validly make the advantageous assumptions that all DMUs have the same technology and pay the same prices for all inputs.

One part of our analysis involves a simple linear (additive) regression model in which total cost was estimated as a function of the three outputs produced by each DMU. The results were as follows:

$$C = -95.300 + \underset{(8)}{152\, y_1} + \underset{(22.2)}{182.4\, y_2} + \underset{(767)}{1302\, y_3}$$

where C = Total cost per year (7)

y_1 = # of regular patients treated per year

y_2 = # of severe patients treated per year

y_3 = Training units provided in one year

1/ Recent literature has begun to supply a variety of means for addressing some of these problems when regression estimates for securing efficiency evaluations are wanted. They do not appear to be very satisfactory, however, and so we do not examine them here. See Banker, Charnes, Cooper and Maindaratta [2]. We confine ourselves only to those types of regressions which have been commonly (and widely) employed. See, e. g., [34].

The standard errors noted in the parentheses below each coefficient indicate high levels of statistical significance. The coefficient signs are positive, as required, and the relation between the y_1 and y_2 (for regular and severe patient) coefficients is in the correct (plausible) direction. A high R^2 value of 0.97 suggests a good fit with the observational data so, by standard reasoning, a high degree of cost variation is "explained" by these independent variables.[1/]

The only apparent discrepancy is a fixed negative cost estimate of \$95,300. This value, which is not statistically significant, might cause the model to be questioned especially in cases involving hospitals with relatively small outputs. Hence another regression with its total cost intercept fixed at zero was calculated. We do not reproduce the results here, however, since (consistent with what has just been said) the resulting coefficient values did not differ greatly from those given in (7). Hence the latter might be used to estimate the incremental cost per unit of each output as in the second column of the following tabulation:

Output	Estimated Incremental Cost	Efficient Incremental Cost	% Deviation
y_1	\$ 152.	\$130	17.0
y_2	\$ 182.40	\$170	7.3
y_3	\$1302.	\$500	160.0

1/ The independent variables were found to have fairly low inter-correlations as follows:

$r_{y_1 y_2} = -0.37$; $r_{y_1 y_3} = -0.03$; $r_{y_2 y_3} = -0.08$.

Focusing on the incremental costs in this manner bypasses the difficulties associated with a negative intercept value. It also corresponds to an assumption (not often stated explicitly) that the slope coefficients may still parallel the true incremental efficiency values, at least roughly, in a manner that corresponds to a shift of the regression plane up to the frontier without altering its slopes.[1/] In the present case, we know the incremental costs for efficient operations and these are supplied in the third column. The estimates from the regression are high in every case. Only the estimate for y_2 (= severe patients) is even tolerable and the estimated cost for y_3 (= teaching) is very wide of the mark.

Another use of such regressions is to evaluate efficiencies as was done by Feldstein [18] in his now classic study of British hospitals. That is the actually observed outputs for each of H1 to H15 would be inserted in an expression like (7) and the resulting total cost would then be compared with the corresponding actual costs at this hospital.[2/] The presence of a negative intercept value could be troublesome, however, and alternate forms of regression functions might then be explored.

1/ This method of parallel-shift treatment is explicitly incorporated in some of the "frontier estimation" methods that have recently been devised. See Forsund, Lovell and Schmidt [19].

2/ A variety of adjustments might be employed to allow for different hospital characteristics and patient mixes, etc. See Feldstein [18] for further discussion.

Another type of function that has been commonly employed in hospital studies, is the so-called Cobb-Douglas form. This form has the advantage of avoiding the possibility of negative intercepts and since, in the present data set, no zero outputs are present for any of the hospitals we can also avoid difficulties that are sometimes experienced from this quarter. Thus we now turn to such a Cobb-Douglas approach.

In logarithmic form our estimated relation obtained from the data of Exhibit 3 is

$$\ln C = 3.98 + \underset{(.04)}{.62} \ln y_1 + \underset{(.07)}{.57} \ln y_2 + \underset{(.05)}{.10} \ln y_3 \tag{8}$$

which, in the usual Cobb-Douglas representation, becomes

$$C = 53.79\, y_1^{0.62}\, y_2^{0.57}\, y_3^{0.10}. \tag{9}$$

In this case the coefficients in (8) and hence the exponents in (9) all appear to be reasonable as well as significant. In sum, however, the exponent values (.62 + .57 + .20) exceed 1 which, being significant, means that evidence of decreasing returns to scale is present, or at least this possibility cannot be rejected. In our case this may reflect the complementary and substitution relations that are known to be present in some of the inputs.[1/] The regression does not detect these relations in this form, however, and the fact that it results in a significant value (with $R^2 = 0.96$) could lead to erroneous recommendations with respect to decisions on the scale of operations.

1/ E. g., as reflected in $A^{-1} x = y$ when going from $x = Ay$, with A a matrix of positive constants as in (1). Thus, in general, A^{-1} will have negative as well as positive elements reflecting relations of complementarity as well as substitution among the various inputs used in producing these output combinations. See Sherman [29] for further discussion.

If we now consider DMUs as potentially inefficient when their actual total cost exceed the estimated total cost in (9), then efficient DMUs H2, H6, and H7 would be erroneously considered inefficient and inefficient DMUs H11, H12, H13, and H14 would be identified as efficient. These results together with the results of our preceding analysis are drawn together and presented in Table 3. In identifying which DMUs are efficient or inefficient, DEA has evidently done better than the others with the exception of the cost ratio approach when the latter is (a) adjusted for case mix and/or (b) identified with "low" and "high" levels of teaching outputs. There is, of course, a degree of arbitrariness present in these cost ratio efficiency and inefficiency characterizations that provide these favorable results for comparison with DEA. Furthermore the Case Mix adjustment procedure we used presupposes a knowledge of the efficient cost of operations and this is reflected in the results shown in both columns (B) and (C) in Table 3. Normally these costs will not be known and so we may count the apparently favorable results of these ratio analyses as proceeding from an assumed knowledge that will generally not be available. This knowledge is not required by DEA and hence we may regard it as being superior to the ratio analysis in these respects as well as in other respects that we shall begin to examine after first summarizing some of our other findings to this point as follows:

1. Ratio (cost) analysis and regression analysis required an arbitrary rule to determine which DMUs would be designated as inefficient. With ratio analysis, the mean might well have been lower or higher depending on whether there were more or fewer efficient units in the data set. Similarly, regression analysis might also have a lower or higher cost curve depending on the relative number of inefficient units.

2. Ratio analysis, as did regression analysis, required price data and other adjustments to address the multiple output and input situation while DEA could address this situation directly. In

addition, the ratios would be confounded if DMUs paid different prices for similar inputs. For example, a DMU that had very low prices might have a lower average cost that could obscure the presence of technical (production) inefficiencies. Regression analysis also assumed DMUs had the same costs/input, and different *unit input costs would have shifted the cost function and could* thereby also conceal inefficiencies.

3. Regression analysis results depended on the selection of an appropriate model or set of cost relationships and nothing in the data set suggested that either of the choices were not appropriate. DEA, however, required no such assumptions.

There are other points that can also be made as we move beyond mere classification into identifying the particular inputs where inefficiencies occur and estimating their amounts. This will be dealt with in the sections that follow.

Table 3

Comparison of DEA, ratio analysis, and linear regression approaches ability to locate Inefficient DMU's

E = DMU rated as efficient
I = DMU rated as inefficient

	(A)	(B)	(C)	(D)
Efficient DMU's	DEA (1) Results	Ratio (2) Analysis	Case Mix Adjusted Average Cost/Patient (3)	Regression (4) (Cobb/Douglas)
H1	E	E	E	E
H2	E	E	E	I
H3	E	E	E	E
H4	E	E	E	E
H5	E	E	E	E
H6	E	E	E	I
H7	E	E	E	I
Inefficient DMU's				
H8	I	I	I	I
H9	I	E	E	I
H10	E	I	I	I
H11	I	E	I	E
H12	I	E	I	E
H13	E	E	E	E
H14	I	E	I	E
H15	I	I	I	I

(1) From table 1

(2) From table 2 column B - DMUs with cost/patient greater than one standard deviation above the mean used to identify inefficient DMUs.

(3) From Table 2 columns C and D with cost/patient greater than one standard deviation above the mean used to identify inefficient DMUs.

(4) Based on rule that DMUs with actual total cost greater than estimated total cost (based on the regression model) are inefficient.

8. Extensions

Perhaps the easiest approach to the topic of identifying the sources and estimating the amounts of inefficiency present in each DMU is to begin with a specific example. We therefore begin with H15 as an illustration of these kinds of additional uses of DEA. This hospital, which is inefficient, has already been discussed in association with H1 in Exhibit 3. We now approach it in a different manner as follows.

First consider the value of $h_o^* = 0.87$ in Table 1. Here we shall use this value to obtain the results shown in the column labelled "Intensity Adjusted Value" in Table 4. Because slack values also need to be considered in assessing efficiency we may refer to these h_o^* values as "intensity factors" and use them in the manner of the $h_o^* = 0.87$ value that is applied to each of the inputs in Table 4. The value which is then obtained in the case of H15 can then be compared with the corresponding value shown under the column labelled "True Efficiency Value". The latter are the values of the inputs actually needed for the outputs of H15 with efficient operations, as obtained from the efficient coefficient values provided in Exhibit 1. The maximum discrepancy of \$(139,200-130,000)= \$9,200 or, approximately, 7% occurs in the case of Supply \$. The other DEA estimates resulting from the intensity adjustment factor applied to the observed inputs are within 2% and 0.3%, respectively, of the true efficiency values.

TABLE 4

H15 INTENSITY ADJUSTMENT AND EFFICIENCY VALUE

	Adjusted Input Values				Efficient Input Values			
					Adjustments			
	Observed Input Value		Intensity Adjustment Factor	Intensity Adjusted Value	Regular	Severe	Teach Units	True Efficiency Value
FTE:	26.5	x	0.87	= 23.055	.004 x 3,000	+ .005 x 2,000	+.03 x 50	= 23.5
Bed Days:	47,370	x	0.87	= 41,211.9	(7 x 3,000	+ 9 x 2,000)	÷ 0.95*	= 41,052
SUPPLY $:	160,000	x	0.87	= 139,200	20 x 3,000	+ 30 x 2,000	+ 200 x 50	=130,000

*0.95 = vacancy factor for efficient production. See assumption (a) in Exhibit 1.

Evidently our h_o^* value has operational significance in that it indicates "*amounts*" *of inefficiency that are present*. It thus differs from the index numbers and like approaches that are sometimes used for efficiency ratings. See, e. g., the index constructed by Feldstein [18] for use in the case of British hospitals.

As indicated earlier, the presence of slack in an optimal tableau is also to be considered a source of inefficiency, and these data, too, are available from the simplex tableaus. In particular, the slack value for Supplies in the optimal solution amounts to $11,880 and 955 Bed Days of slack are also present. When these amounts are subtracted from the Intensity Adjusted Values in rows 3 and 2 of Table 4 new estimates for efficient inputs in these factors become $127,313 and 40,257 BD, respectively. This greatly improves the efficiency estimate of the former along with some worsening of the latter. All estimates are now within about 2% of the true efficiency value.

It is not contended that DEA efficiency estimates will always be this close and, indeed, reference to Table 5 will show estimates that are very wide of the mark for H10 in at least 2 of the 3 pertinent input categories. On the other hand, even in this case the estimates are both better and more detailed than those obtained from the ratio and regression approaches discussed earlier in this article. Also, as was observed in our discussion of Table 1, there are strong reasons to suspect the $h_o^* = 1$ intensity values for H10 and H13. Elimination of these two hospitals still leaves H11 with errors in the range of 10-15% for three of the input estimates, while all of the other errors are in a range of about at 2% or less. Furthermore this record is considerably improved when the efficient hospitals, H1 to H7, are added to the list since in their case the estimates all have zero errors.

This seems to be a very creditable performance, at least compared to what the other approaches appear to offer for use on the data base we have erected. Further testing will also be required both on other data bases and in actual uses, of course, and improvements in the methodology and alternate modeling approaches and estimation methods will also need to be explored.

Methods by which such testing might be done will be discussed in the next section. We can then conclude this section by noting that still other uses of DEA are also possible. For instance, what we have been doing in this section amounts to projecting each DMU onto the relevant position of the efficiency surface in conformance with the methods prescribed in [13]. Further tradeoffs may then be effected by reference to the marginal rates of transformation and/or substitution via the optimal u_r^* and v_i^* values

which may be secured from the simplex tableaus. See (6). These values can provide guidance for augmenting or contracting the inputs and outputs of the corresponding DMU and, at the same time, provide controls and guidance on efficient uses by the managers of these DMUs.

These u_r^* and v_i^* values will represent estimates which, of course, may not be wholly accurate. The same is true of the similar uses of regression estimates but, in addition, such regression estimates can be expected to be very wide of the efficiency values--as should be clear from our earlier discussions. Indeed, as noted in [2], the estimates of such substitution and transformation rates generally continue to be very far from the true efficiency values even when the simple forms of regression functions used in the present article are replaced by more general and flexible forms and when the statistical methods used are specifically directed toward frontier efficiency estimates.

TABLE 5

ESTIMATED AND TRUE EFFICIENCY VALUES

H8 to H 15

HOSPITAL	INPUTS	OBSERVED VALUE	INTENSITY ADJUSTED VALUE	SLACK	ESTIMATED EFFIC. VALUE	TRUE EFFIC. VALUE	% DIFF.
H8	FTE	25.0	24.75	--	24.75	25.0	1.0
	BD	49,475	48,980	8,425	40,555	41,053	1.2
	$S	140,000	138,600	--	138,600	140,000	1.0
H9	FTE	24.5	24.01	--	24.01	24.5	2.0
	BD	43,160	42,297	--	42,297	43,158	1.9
	$S	165,000	161,700	25,000	136,700	140,000	2.4
H10	FTE	77.0	77.0	--	77.0	53.0	45.0
	BD	92,630	92,630	--	92,630	92,632	0.0
	$S	340,000	340,000	--	340,000	280,000	21.4
H11	FTE	44.5	37.8	5.1	32.7	36.5	10.4
	BD	65,260	55,471	--	55,471	65,263	15.0
	$S	265,000	225,250	45,711	179,539	200,000	10.2
H12	FTE	30.0	29.7	--	29.7	30.0	1.0
	BD	60,000	59,400	9,476	49,924	50,526	1.2
	$S	170,000	168,300	--	168,300	170,000	1.0
H13	FTE	43.5	43.5	--	43.5	43.5	0.0
	BD	81,110	81,110	--	81,110	76,842	5.6
	$S	245,000	245,000	--	245,000	240,000	2.1
H14	FTE	30.0	29.7	--	29.7	30.0	1.0
	BD	60,000	59,400	9,476	49,924	50,526	1.2
	$S	170,000	168,300	--	168,300	170,000	1.0
H15	FTE	26.5	23.06	--	23.06	23.5	1.9
	BD	47,370	41,212	955	40,256	41,053	1.9
	$S	160,000	139,200	11,887	127,313	130,000	2.1

Note: H10 and H13 have intensity values of $h_o^* = 1$.

9. Conclusion

The really surprising result is not how well DEA performed on our manufactured data base, but rather the poor performance of the econometric-statistical models we employed. These models are representative of many analyses that have been employed in studies used to draw important policy conclusions. Two recent multi-million dollar studies of this kind that resulted in multi-volume reports with important findings for policy formation are: (1) U.S. Department of Health, Education and Welfare, PSRO: An Initial Evaluation of the Professional Standards Review Organization [in Health Care Delivery][1/] and (2) U.S. Office of Education, The Follow Through Planned Variation Experiment [for Education of Disadvantaged Children].[2/]

The questions raised by our across-DMU regression results would seem to apply *a fortiori* to studies like these since in our case the design of the data base was favorable to assumptions such as a common technology and a common price structure across the DMUs. Assumptions like these are much less likely to be valid for regressions used in applied studies, such as the kinds we just cited.

It might be argued that it is unfair to level criticisms such as these at regression models designed to handle only one dependent variable at a time and using methods of estimation directed toward average rather than efficient behavior.[3/] In the study [2], which we conducted with R. Banker and A. Maindiratta, however, both of these qualifications were accommodated.

1/ See [32]. See also [17] for further discussion and suggestions for alternative approaches.

2/ See [33]. See also [12] for further discussion and suggested alternative approaches.

3/ Note, however, the study by Feldstein [18] which was conducted in just this manner and numerous other studies of this type can also be cited. See also the study by Banker, Conrad and Strauss [4] which consisted of a DEA redo of a previously conducted econometric study of North Carolina hospitals (using a translog function) and arrived at drastically different conclusions on the presence of returns to scale, etc., which had been found not to be present in the original (econometric) study.

In that study, conducted in the same spirit as the one we are presently summarizing, a piecewise Cobb-Douglas function with one output as the dependent variable was used to represent a continuous technology with increasing and decreasing returns to scale in its various segments. Technical as well as scale inefficiencies were then introduced into randomly generated observations as a basis for comparing DEA with so-called flexible functional form approaches using translog regressions. DEA again performed very well but, perhaps even more importantly, the statistical-econometric approaches performed poorly--not only relative to DEA but also in a manner that was unsatisfactory *per se*--in both technical and scale efficiency identification and estimation. Moreover, the estimation methods employed for the regressions in this case were of the so-called "corrected least squares" varieties, as specifically designed for the purpose of locating and estimating efficiency frontiers. See [26] and [19].

One possible source of trouble, we think, lies not merely in the estimation methods but rather in an approach--the one that is commonly taught and employed--which tries to capture a great variety of behaviors in only relatively smooth and simple (e. g., unconstrained) functional forms. Attempts to meet these difficulties by weighted regressions, outlier analyses and similar approaches do not really deal with the problem in a sufficiently fundamental way, we think, and other alternatives need to begin to be considered.

The optimizations involved in these DEA and statistical approaches also need to be considered. Generally speaking the commonly employed statistical approaches optimize over *all* observations while DEA optimizes relative to *each*. Another way of stating this is to note that a complete DEA analysis will, in general, involve n optimizations, one for each

observation, while the usual statistical approach involves only one.

This implies that differences in testing for results and checking for possible inferences must also be expected. Because it is directed toward individual observations, DEA is also directed to each DMU in a way which suggests this as a fundamental unit of test. That is, the inferences that are made about at least some of these DMUs can and should be tested by on-site observations in ways, and with results, that differ from testing statistical estimates for general types of class properties effected across all observations.

Emphasis on individual observations is one way to distinguish DEA from customary types of analyses. But DEA is directed to relative efficiencies so that comparative analyses of subsets of the observations are also in order. Here, too, however, differences from customary approaches need to be noted.

Consider, for instance, the estimates of marginal rates of transformation and/or substitution that are associated with each (efficient) reference surface.1/ As developed in [13],2/ these values and the related (efficient) marginal rates of substitution are available from the dual variable values which are to be found in the optimal simplex tableaus.

All DMUs which have the same efficient reference facet will have the same dual variable (= marginal rate of transformation)3/ values. What this means can be highlighted by contrasting these estimated values with those which are available from ordinary statistical regression approaches applied to individual DMUs. The latter can also be used to obtain estimates of such marginal rates of transformation, but these values refer only to the behavior

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1/ In the case of single outputs these would be identified as isoquants and the marginal rates of transformation would become the marginal productivities and the marginal rates of substitution that could then also be derived from these estimates. See [13].

2/ See also [25].

3/ As explained in [11], these are better named "virtual transformation rates."

of each such DMU whereas the DEA values are obtained from the efficient reference set. These values provide the efficient marginal rates of substitution and transformation for evaluating the effects of possible resource augmentations and reductions. The estimates that are thus obtained from DEA indicate what will (or should) be done with efficient usages of these augmentations or reductions--and this may differ greatly from the past behavior of inefficient DMUs.

As noted earlier in this article, computer codes that are already available will provide a printout that identifies the efficient DMUs for each relevant facet. The means for effecting a comparative analysis is then conveniently at hand. Convenient ways for assessing the sensitivity of these efficient DMUs to data variations have also now been developed. See [10]. A way is thereby opened for both immediate use and further research on DEA approaches to areas such as, e. g., the behavior of DMUs in the not-for-profit sector that have proved so resistant to other types of approaches.

Having identified these differences and their possible separate avenues of application, testing and research, we can probably best close on a somewhat different note by indicating ways in which the two approaches might be joined together. One possibility is to use each approach, regression or ratio analysis and DEA, to check on or fortify the other.[1/] Other possibilities exist, however, which might briefly be sketched as follows.

Aigner and Chu in [1], essayed a new approach to frontier estimation by means of what would now be called "goal programming"[2/] with only one-sided deviations permitted so that, in general, the estimated production

1/ See [12] for further discussion on different conditions which might lead to one approach or the other in complementary fashion for policy guidance purposes.

2/ This was originally referred to as "inequality constrained regressions." See [10] and [8]. Although not available at the time of the Aigner-Chu work [1] we would now add the further possibilities that are now available from the goal interval programming approaches described in [9].

function (e. g., a Cobb-Douglas form) would lie on or above all of the observed output values. Confining all deviations to one side clearly does not exhaust the possibilities, however, and one may go on to prescribing proportions of the total deviations or even deviations for individual observations that must lie on one side or the other of an estimated frontier.

In a similar spirit, C. P. Timmer in [30] used "chance constrained programming" formulations and concepts to effect efficiency estimates. Instead of utilizing the power of chance constrained programming, e. g. to deal with different proportions and even different probability distributions, constraint by constraint, Timmer proceeded in an entirely different direction and in the spirit of a "global" statistical analysis discarded "outlier" observations one after another until he achieved what he regarded as "stable" estimates. Notice, however, that this procedure is one which obliterates a great deal of information. In particular, in pursuit of one global (overall) property,[1/] it discards <u>efficient</u> DMUs without even bothering to investigate them individually.

The approaches by Aigner and Chu [1] and by Timmer [30] that we have just described involve a use of inequality constrained optimizations, to be sure, but they otherwise proceeded in the spirit of classical statistical approaches. Something more may also be accomplished along these latter lines. For instance, one might use a discriminant-function or cluster-analytic approach to locate subsets of the original points which have different properties. Hopefully this could include clusters or discriminant subsets of efficient and inefficient points. Separate regressions fitted to these subsets might then yield improved ways of identifying inefficiencies and estimating their amounts.

1/ This is contrary to the spirit of individual observation investigation that we urged, above, and for which the kind of stability analysis provided in [10] is now available.

We have not investigated the latter types of topics, as we shall do in future papers, for the simple reason that we sought to adhere as closely as possible to the kinds of approaches that have generally been used in the kinds of studies we have been considering. Notice that a use of the discriminant and/or cluster analysis approaches we have just described involves an estimation of more than one regression relation and more than one optimization. The other approaches of global programming and chance constrained programming varieties, as in Aigner and Chu [1] and Timmer [30], involve inequality constrained relations of a kind that are similar to the ones used in DEA. Thus, we conclude that there are additional avenues of possible relations between DEA and these other approaches that also invite exploration.

Exhibit 1 Appendix -

Structural Model (Efficient Hospital Operations): Efficient Input-Output and Cost Relationships Assumed in the Hospital Production Model to Create the Simulated Data Base in Exhibit 2

Outputs:	Amount of each Input required to Efficiently produce one unit of output.			Efficient Costs of Outputs
	Full Time Equivalents of Labor (FTE's)	Bed days available	Supply $'s	
Regular Patient Care	.004 FTE/per patient	7 beddays/per patient	$20 supplies/per patient	$130/regular (1) patient
Severe Patient Care	.005 FTE/per patient	9 beddays/per patient	$30 supplies/per patient	$170/severe (2) patient
Training outputs	.03 FTE/per training unit	-	$200. supplies/per training unit	$500/training (3) unit

Input $ Costs for all Hospitals

FTE's	Beddays	Supply's
$10,000/FTE	$10/Bed day	$1/Supply unit

Other Assumptions:

a) Vacancy Rate - Efficient hospitals will have 5% of total beds vacant during the year (available for emergencies).

b) There are no regional cost differences for bed days, supplies, and FTE's and that the mix of FTE's and supplies are similar between hospitals.

c) Cost of unused bed days = $10/bed day.

(1) Cost/regular patient = (.004 FTE/patient)($10,000/FTE + (7 bed days/patient) ($10/bed day) + ($20 supplies/patient = $130/patient

(2) Cost/severe patient = (.005 FTE/patient) ($10,000/FTE) + (9 bed days/patient) ($10/bed day) + ($30 supplies/patient) = $170/patient

(3) Cost/training unit = (.03 FTE/training unit) ($10,000/FTE) + ($200 supplies/training unit) = $500/patient

Appendix

Exhibit 2

Constructed Data Base

(circled) = Inefficient use of inputs compared to a structural model of efficient input-output relationship described in Ex. 1

	Inputs					Outputs			Actual Inputs Used								
									FTE		Bed days		Supply $		Training		
	FTE	Bed Days	Supply $'s	Total* Pat.s	% Sev.** Pat.s	Teach. Units	Reg. Pat.s	Sev. Pat.s	Reg. Pat.s	Sev. Pat.s	Reg. Pat.s	Sev. Pat.s	Reg. Pat.s	Sev. Pat.s	Supply $'s	FTE	Vacancy Rate
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)
H1	23.5	41050	$130,000	5000	40	50	3000	2000	.004	.005	7	9	20	30	200	.03	.05
H2	24.5	43160	140,000	5000	60	50	2000	3000	.004	.005	7	9	20	30	200	.03	.05
H3	26.0	43160	150,000	5000	60	100	2000	3000	.004	.005	7	9	20	30	200	.03	.05
H4	25.0	41050	140,000	5000	40	100	3000	2000	.004	.005	7	9	20	30	200	.03	.05
H5	28.5	50530	160,000	6000	50	50	3000	3000	.004	.005	7	9	20	30	200	.03	.05
H6	36.0	62105	210,000	7000	71	100	2000	5000	.004	.005	7	9	20	30	200	.03	.05
H7	51.5	92630	270,000	12000	17	50	10000	2000	.004	.005	7	9	20	30	200	.03	.05
H8	25.0	49475	140,000	5000	14	100	3000	2000	.004	.005	9	10	20	30	200	.03	.05
H9	24.5	43160	165,000	5000	60	50	2000	3000	.004	.005	7	9	25	35	200	.03	.05
H10	77.0	92630	340,000	12000	17	100	10000	2000	.006	.007	7	9	25	35	200	.03	.05
H11	44.5	65260	265,000	8000	38	50	5000	3000	.005	.006	7	9	30	35	200	.03	.05
H12	30.0	60000	170,000	6000	50	100	3000	3000	.004	.005	7	9	20	30	200	.03	.20
H13	43.5	81110	245,000	9000	56	50	4000	5000	.004	.005	7	9	20	30	300	.05	.10
H14	30.0	60000	170,000	6000	50	100	3000	3000	.004	.005	9	9	20	30	200	.03	.10
H15	26.5	47370	160,000	5000	40	50	3000	2000	.005	.005	9	9	30	30	200	.03	.05

* Total Patients = Col. 7 + Col. 8

** % Severe = Col. 8 ÷ Col. 4

Note: Outputs (Col.s 4 - 8) and input usage (Col.s 9 - 17) are arbitrarily assigned so that H1 - H7 are efficient based on the production model in Ex. 1 and H8 - H15 are inefficient based on the same model. Inputs (Col. 1, 2, 3) are derived from Col.s 4 - 17 as indicated in Ex. 3

Exhibit 3 Appendix -

Example of Construction of Data Base for Hospitals H1 (efficient) and H15 (inefficient)

	H1 Efficient	H15 Inefficient	Difference Between H1 and H15	Inputs Required for Each Unit of the Related Output: FTE/yr. H1		FTE/yr. H15	Bed days/yr. H1		Bed days/yr. H15	Supply $'s/yr. H1		Supply $'s/yr. H15	Vacancy* Rate during the year. H1	Vacancy* Rate during the year. H15
Outputs (identical for H1 and H15)														
Y_1 Regular patients/yr.	3000	3000	-	.004	≠	.005	7	≠	9	$20	≠	$30	5%	5%
Y_2 Severe patients/yr.	2000	2000	-	.005	=	.005	9	=	9	30	=	30		
Y_3 Teaching units/yr.	50	50	-	.03	=	.03	-		-	200	=	200		
(outputs are identical for H1 and H15)														
Total Inputs Required														
FTE	23.5 (1)	26.5 (2)	3											
Bed days	41,050 (3)	47,370 (4)	6,320											
Supplies	$130,000 (5)	$160,000 (6)	$30,000											
Total Cost	$775,500 (7)	$898,700 (8)	$123,200											

*Vacancy rate reflects the % of vacant beds that exist. Beds available to exceed the number used by 1/.95 in efficient hospitals (5% are vacant). If 7 beds are used, 7/.95 must be available on average during the year.

(1) H1-FTE's = (3000 Reg. Pat.)(.004) + (2000 Sev. Pat.)(.005) + (50 Teach. Units)(.03) = 23.5

(2) H15-FTE's = (3000 Reg. Pat.)(.005) + (2000 Sev. Pat.)(.005) + (50 Teach. Units)(.03) = 26.5

(3) H1-Bed days = [(3000 Reg. Pat.)(7) + (2000 Sev. Pat.)(9)] ÷ (.95 Vacancy factor) = 39,000 ÷ .95 = 41,050

(4) H15-Bed days = [(3000 Reg. Pat.)(9) + (2000 Sev. Pat.)(9)] ÷ (.95 Vacancy factor) = 45,000 ÷ .95 = 47,368

(5) H1-Supply $'s = (3000 Reg. Pat.)(20) + (2000 Sev. Pat.)(30) + (50 Teach. Units)(200) = $130,000

(6) H15-Supply $'s = (3000 Reg. Pat.)(30) + (2000 Sev. Pat.)(30) + (50 Teach. Units)(200) = $160,000

(7) H1-Total Cost = (23.5 FTE)($10,000/FTE) + (41,050 Bed days x $10/bed day) + $130,000 Supplies = $775,500

(8) H15-Total Cost = (26.5 FTE)($10,000/FTE) + (47,368 Bed days x $10/bed day) + $160,000 Supplies = $898,700

REFERENCES

[1] Aigner, D. T. and S. F. Chu, "On the Estimation of the Industry Production Function," American Economic Review, Vol. 55, No. 4, September 1968, pp. 826-839.

[2] Banker, R. D., A. Charnes, W. W. Cooper and A. Maindiratta "A Comparison of DEA and Translog Estimates of Production Frontiers Using Simulated Observations from a Known Technology," Research Report, Center for Cybernetic Studies, The University of Texas at Austin, October, 1982.

[3] ___________ ___________ ___________ "Estimation of Technical and Scale Efficiencies in Data Envelopment Analysis," Management Science, (forthcoming).

[4] ___________, R. Conrad, and R. Strauss, "An Application of Data Envelopment Analysis to A Study of Hospital Efficiencies and Scale Sizes," Working paper 81-9, Carnegie-Mellon University, December, 1981.

[5] Bessent, A. and W. Bessent, "Determining the Comparative Efficiency of Schools Through Data Envelopment Analysis," Research Report CCS 361 (Austin, Texas: Center for Cybernetic Studies, The University of Texas at Austin, December 1979).

[6] ___________ ___________, J. Kennington and B. Regan, "An Application of Mathematical Programming to Assess Productivity in the Houston Independent School District," Management Science (forthcoming).

[7] Charnes, A. and W. W. Cooper "Goal Programming and Constrained Regression," Omega 3, No. 4, 1975, pp. 403-404.

[8] ___________ ___________ "Goal Programming and Multiple Objective Optimizations," European Journal of Operational Research 1 No. 1, 1977, pp. 39-54.

[9] ___________ ___________ and R. O. Ferguson, "Optimal Estimation of Executive Compensation by Linear Programming," Management Science 1 No. 2, 1955, pp. 138-151.

[10] ___________ ___________ A. Lewin, R. Morey and J. Rousseau, "An Approach to Positivity and Stability Analysis in DEA" Management Science (submitted).

[11] ___________ ___________ and E. Rhodes "An Efficiency Opening for Managerial Accounting in Not-for-Profit Entities" in H. P. Holzer, Ed., Management Accounting 1980: Proceedings of the University of Illinois, Management Accounting Symposium (Urbana: University of Illinois, 1980).

[12] ___________ ___________ ___________ "Evaluating Program and Managerial Efficiency: An Application of Data Envelopment Analysis to Program Follow Through," Management Science, Vol. 27, No. 6, June 1981, pp. 668-697.

[13] ___________ ___________ ___________ "Measuring Efficiency of Decision Making Units," European Journal of Operational Research, Vol. 3, No. 4, July 1979, p. 339.

References (Continued)

[14] ______ ______ ______ "Short Communication: Measuring Efficiency of Decision Making Units," European Journal of Operational Research, Vol. 3, No. 4, July 1979, p. 339.

[15] ______ ______ ______, L. Seiford and J. Stutz, "A Multiplicative Model for Efficiency Analysis," Socio-Economic Planning Sciences (forthcoming).

[16] ______ ______ ______ ______ "Invariant Multiplicative Efficiency and Piecewise Cobb-Douglas Envelopments," Operations Research Letters (forthcoming).

[17] Churchill, N. C., W. W. Cooper and V. Govindarajan, "Effects of Audits on the Behavior of Medical Professionals Under the Bennett Amendment," Auditing: A Journal of Practice and Theory Vol. 1, No. 2, Winter 1982.

[18] Feldstein, M., Economic Analysis for Health Service Efficiency (Amsterdam: North-Holland Press 1968).

[19] Forsund, F. R., C. A. K. Lovell, and P. Schmidt, "A Survey of Frontier Production Functions and of Their Relationship to Efficiency Measurement," Journal of Econometrics, Vol. 13, No. 1, 1980, pp. 5-26.

[20] Haney, W. "The Follow Through Planned Variation Experiment," The Follow Through Evaluation: A Technical History, Vol. 5. Prepared for the Office of Planning, Budgeting and Evaluation of the U.S. Education Department of HEW by the Huron Institute, Cambridge, Mass., 1977.

[21] Lewin, A. Y. and R. C. Morey, "Evaluating the Administrative Efficiency of Courts," Omega (forthcoming).

[22] ______ ______ "Studies to Validate and Extend the Data Envelopment Analysis Method for Measuring Relative Efficiency of Decision Making Units," Durham, N. C. Duke University, Fuquaa School of Business, 1980.

[23] Massachusetts Rate Setting Commission, Development of a Budget Approval Program for Massachusetts Hospitals, 1976-1978, Final Report, 12/1/78, HCFA contract #600-76-0174, Vol. 2, Chapter 8 (Washington, D. C. HCFA, 1978).

[24] ______ ______ ______ ______ Hospital Bureau, "Grouping of Hospitals for Purposes of Determining Reasonable Cost Levels," Document No. 114.1 CMR, Rate Setting Commission, 1980.

[25] Rhodes, E., Data Envelopment Analysis and Related Approaches for Measuring the Efficiency of Decision Making Units with an Application to Program Follow Through in U. S. Education Ph.D. Thesis (Pittsburgh: Carnegie-Mellon University, School of Urban and Public Affairs, 1978).

References (Continued)

[26] Richmond, J., "Estimating the Efficiency of Production," International Economic Review, 1974, pp. 515-521.

[27] Sato, D., Production Functions and Aggregation (Amsterdam: North Holland Publishing Co., 1975).

[28] Schinnar, A. P. "Frameworks for Social Accounting and Monitoring of Invariance, Efficiency and Heterogeneity," Feis Discussion Paper 147 (Philadelphia: University of Pennsylvania, School of Public and Urban Policy, 1980).

[29] Sherman, H. D., Measurement of Hospital Technical Efficiency: A Comparative Evaluation of Data Envelopment Analysis and Locating Inefficiency in Health Care Organizations," unpublished Doctoral Dissertation, Boston; Harvard University Graduate School of Business Administration, 1981.

[30] Timmer, C. P. "Using a Probabilistic Frontier Production Function to Measure Technical Efficiency," Journal of Political Economy LXXIX No. 4., July-Aug., 1971, pp. 776-794.

[31] U.S. Congressional Budget Office Background Paper, The Effect of PSROs on Health Care Costs: Current Findings and Future Evaluations (Washington: U.S. Government Printing Office, June, 1979).

[32] U.S. Department of Health, Education and Welfare, PSRO: An Initial Evaluation of the Professional Standards Review Organization Vol. 1, Executive Summary, Washington, 1978, National Technical Information Service II PB 278154/AS, HEW Report OPEL 77-12, Feb., 1978.

[33] U.S. Office of Education, The Follow Through Planned Variation Experiment, Vols. IIA, IIB, IIC. Washington, 1977.

[34] U.S. General Accounting Office, "Problems with Evaluating the Cost Effectiveness of Professional Standards Review Organizations," Report by the Comptroller General of the United States HRD-79-52 (Washington: U.S. General Accounting Office, July, 1979).

[35] Zaretsky, H. W. "The Effect of Patient Mix and Service Mix on Hospital Costs and Productivity," Topics in Health Care, A. Herkimer, Ed., Vol. 4, No. 2, Winter 1977, pp. 63-82.

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Efficiency, hospitals, data envelopment analysis, statistical regression, ratio analysis, simulation, validation

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Data Envelopment Analysis (DEA), a new methodology based on mathematical programming models, provides an approach to evaluation of the relative efficiency of organizations, especially not-for-profit organizations which have multiple outputs and inputs. This paper uses an artificial data base to evaluate DEA relative to other alternatives such as ratio and regression analyses. The results of this study generally favor DEA not only for identifying inefficiencies but also for locating their sources and

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20. ABSTRACT (continued)

estimating their amounts in particular DMUs (Decision Making Units). Statistical regressions performed very poorly, per se, as well as by comparison. Reasons for the poor performance of these customary statistical regression approaches are indicated along with possible ways of improving this performance.